Applying Calculus to an Oil Spill

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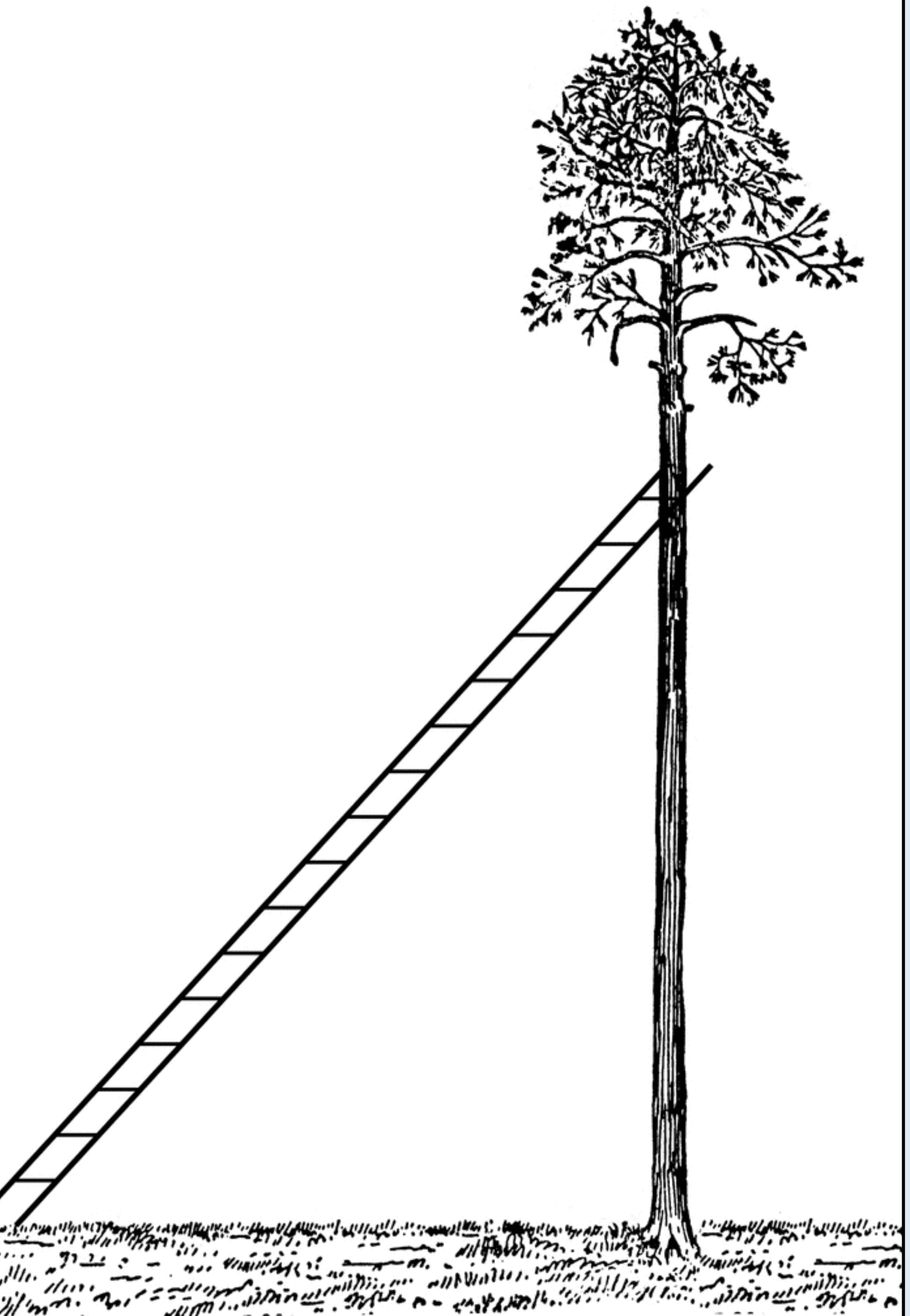

Ladder, 2007

A Quick Introduction to Related Rates

- Related rates, as the name implies, have to do with finding the rate at which a quantity changes by relating it to other quantities whose rates of change are known (Kreider et al., 2005). This makes it possible to find rates of change that might be very difficult to find otherwise.
- Common examples are the lengths of the sides of the triangle a ladder makes as it slides down a tree, or the elongation of a shadow over the course of the day.

The Problem

"An explosion at an oil rig located in gulf waters causes an elliptical oil slick to spread on the surface from the rig. The slick is a constant 9 in. thick. After several days, when the major axis of the slick is 2 mi long and the minor axis is ¾ mi wide, it is determined that its length is increasing at the rate of 30 ft/hr, and its width is increasing at the rate of 10 ft/hr. At what rate (in cubic feet per hour) is oil flowing from the site of the rig at that time?" (Hass et al., 2020)

Strategy

1. Draw a picture and name the variables and constants.
2. Define variables.
3. Identify what you have been asked to find.
4. Write an equation that relates the variables.
5. Differentiate with respect to t.
6. Evaluate.

(Hass et al., 2020)

Solving the Problem

Known Values:

$$Major\ axis = a = \frac{2\ mi}{2} = 5280\ ft \qquad h = 9\ in = 0.75\ ft$$

$$Minor\ axis = b = \frac{3}{4} mi \cdot \frac{1}{2} = 1980\ ft$$

$$\frac{da}{dt} = 30\ \ ft/hr \cdot \frac{1}{2} = 15\ \ ft/hr \qquad \frac{dh}{dt} = 0\ ft/hr$$

$$\frac{db}{dt} = 10\ \ ft/hr \cdot \frac{1}{2} = 5\ \ ft/hr$$

Unknown: $\frac{dV}{dt}$

Equations for Volume and Area:

$$V = h \cdot \pi ab \qquad A = \pi ab$$

$$V = h \cdot A$$

Differentiate:

$$\frac{dV}{dt} = A\frac{dh}{dt} + h\frac{dA}{dt} = h\frac{dA}{dt} = h\pi \cdot (a\frac{db}{dt} + b\frac{da}{dt})$$

Evaluate:

$$\frac{dV}{dt} = \pi \cdot 0.75\ ft \cdot (5280\ ft\ \ \cdot 5\ \ ft/hr + 1980\ ft \cdot 15\ \ ft/hr)$$

$$\frac{dV}{dt} = \pi \cdot 0.75\ ft \cdot (56100\ \ ft^2/hr)$$

$$\frac{dV}{dt} = \pi \cdot 42075\ ft^3/hr$$

$$\frac{dV}{dt} \cong 132183\ \ ft^3/hr$$

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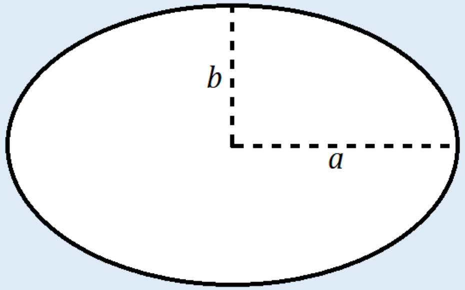
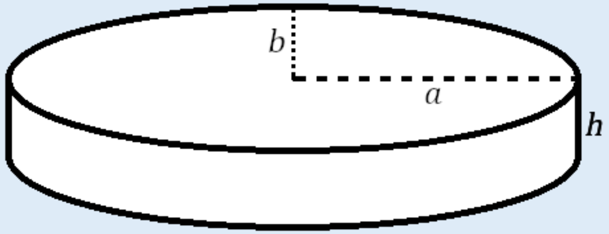


$A=\pi ab$



$V=h\cdot\pi ab$

Conclusion and Other Applications of Related Rates

- Obviously, a real oil spill would not be perfectly elliptical, nor would it have a constant depth. However, treating it as a cylinder makes it much easier to estimate its rate of expansion. Quickly estimating the severity of an oil spill is crucial for both containment and cleanup.
- Related rates are one of the clearest and most wide-reaching applications of calculus. They make frequent appearances in science and engineering, but they can also be applied to DIY projects, cooking, and other day-to-day activities.

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Citations

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